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ABSTRACT

The linear programing approach to position-salary evaluation of school district personnel allows the district to calculate a system of relative weights that establish the relationship of one position to another in quantitative terms. The model provides an internally consistent, effective assessment of an individual's relative worth to the school district in terms of salary. Linear programing models provide fairly sophisticated methods for efficient allocations of resources consistent with the objectives and constraints of the school district. A related document is EA 003 594. (Author/RA)

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A LINEAR PROGRAMMING APPROACH TO POSITION-SALARY EVALUATION IN SCHOOL PERSONNEL ADMINISTRATION

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PREFACE

How much should a person with a given set of qualifications be paid so that his salary is consistent with other salaries in an organization? The purpose of this document is to explore the possible utilization of linear programming techniques for answering the above question in job-salary evaluation. Specifically, a linear programming model will be used to analyze a hierarchical salary structure in a school district. The model will be used to determine the relative importance of each of the compensable elements of each function (position) in a school district. In addition, monetary equivalents to these elements, will be derived from the model in order to develop a consistent scale of compensation within a school district. The proposed type of job-salary evaluation scheme might have wide application in the Air Force as well as civilian sector, since it could be of important use in determining discrepancies or in determining those positions in the Air Force which seem "out of line" in terms of salary or grade classification. The development of utility objective functions, so factors of a job which are considered crucial, receive higher relative weights would tend to increase the flexibility of the salary schedule in responding to the changing needs and demands of the organization. The Air Force might use this job-evaluation scheme for evaluating the GS classification for a set of jobs and identifying those jobs in which the GS classification is not consistent with the skills, and knowledge demanded by the job (e.g., the job is over- or underrated salarywise). The process of developing the

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job-evaluation scheme would also assist the Air Force in the development of valid job descriptions and determining the qualification necessary for the job. Finally, this evaluation scheme could also be useful in identifying those jobs in the organizational hierarchy where present personnel are either over or under qualified. For illustrative purposes only, the hierarchical salary structure for a school district organization was chosen for analysis.

I. INTRODUCTION

The use of linear programming models for solving resource allocation problems is well established in the literature of business and industry. Only recently has linear programming been used for a resource allocation problem in education.(1) The purpose of this document was to explore the application of linear programming to a position-salary evaluation scheme for a local school district. This study could be of great practical value for those school districts which are desirous of producing salary schedule which reflect both the economic demands for the particular skill possessed by the teacher (e.g., science or math teacher) and allows for fair and equitable renumeration for school district personnel commensurate with these contributions to the accomplishment of the objectives of the school district.

The traditional fixed-step salary schedule, common to most school districts in America, has led to widespread discontent by both school district administrators, who have to compete against the private sector for personnel possessing certain desired skills and knowledge, and by teachers who complain that their salary does not reflect the demands placed upon them by the difficulty of the learning situation (e.g., teaching in a ghetto school). This disparity in the learning environment is usually not reflected in the current teacher salary schedules. In a recent Los Angeles newspaper article addressing itself to the problems of the cities it was stated:(2)

> Teachers in Los Angeles are given no salary differential. Not so surprisingly, then most of them choose to work in places like Westwood or Sherman Oaks. The result has been a serious decline in teacher quality in areas where teacher quality should be the highest. One answer would be incentives for teachers in ghetto schools. This would bring better talent into these classrooms and would work to the advantage of other problems brought on by a uniform salary schedule (e.g., shortages of teachers of English, mathematics and sciences and surpluses of teachers of art, social studies and physical education. A differential wage, tried to supply and demand conditions, could give the L.A. system a better balance of education.

Some of the major problems associated with the quality of education have been associated with teacher salaries. McKean and Kershaw believed that the structure of teacher salaries is a key both to understanding and the nature and causes of teacher shortages, and in solving some of the major problems associated with cost and quality in public education.[3]

Fawcett[4] and other educators[5,6] have proposed that teacher salaries be commensurate with the skills, attitudes and knowledge required by the school district.

In the literature of school personnel administration, "merit pay" schemes or schemes for evaluating and financially rewarding teachers both in times of the difficulty of the learning situation and the quality of the teaching have been proposed.[7,8] Unfortunately, the difficulties in developing objective measures of teacher quality have severely limited the development of differential salary plans for school districts. Various programs have been developed in some of the more affluent school districts throughout the country. One of the most promising differential salary schemes is the incentive increment program. The rationale for the incentive increment program is to elevate the quality of teaching by offering incentive increments in salary to those teachers in a school district who desire to undergo a fairly long-term professional growth or improvement program. In the incentive increment program the teacher, in conjunction with the department head and the school administration, outlines a detailed plan for professional growth (e.g., college classes, curriculum development, writing a book, etc.). The successful completion of this mutually agreed upon program entitles the teacher to an increment in salary. A more detailed discussion of the incentive increment program can be found in Ref. 11 in the bibliography. The shortcomings of the incentive increment programs are (1) the teacher must have tenure before becoming eligible to participate in the program, (2) there is no recognition in the program for differences in academic background, difficulty of the learning situation or the quality of the teaching. In short, the incentive increment program does not recognize the individual or teacher's "worth" to the school district and it is severely limited in scope (only applies to tenured teachers).

The criteria to be used in judging an individual teacher's worth to the school district would depend, to a great extent, upon the stated objectives of the school district. Some objectives of a school district might be getting as many students as possible into college, another might be socialization or citizenship. While the abovementioned objectives are not mutually exclusive sets, nevertheless, most schools would consider some objective to be of greater importance than others. In accordance with the objectives of the school district, a teacher with a PhD in a specific learning situation might be considered to be of more relative worth than a teacher with a BA. The primary problem and the one to which this paper will address itself is exactly how much more. Certainly one important criteria of a teacher's worth to the organization should be the quality of his teaching. The evaluation of teacher performance might in the past be measured by the academic success of his students or by students' evaluations. Since many of the certified school district personnel (counselors, administrators, etc.) are not directly involved in teaching students, the criteria of teacher quality will not be considered in the discussion. Instead, an analysis of the hierarchical salary structure of a school district will be the main focus of the paper.

II. PURPOSE

The purpose of this study is to formulate a linear programming model for a position and salary evaluation for certified school district personnel (teacher aids, teachers, department heads, school administrators) in a school district. Notice some school district certified personnel such as counselors, psychometrists, etc., will not be considered in the analysis, since there is some overlapping of responsibilities and function, e.g., some counselors teach one or two classes and in some instances special types of certified personnel are paid at an hourly rate.

Recently, research has been completed in business and industry into the use of linear programming techniques as a means for determining executive compensation and job-salary evaluation.[9] Since

salaries constitute the major budgetary item in school districts and these are increasing teacher demands for more equitable pay differentials between various functions or positions (e.g., administrators, teachers, etc.) in a school district, the problem is considered worthy of investigation both from a theoretical and practical viewpoint.

III. PRELIMINARY CONSIDERATIONS IN DEVELOPING THE MODEL

Before an effective position-salary evaluation scheme for school district personnel can be developed a great deal of time and effort on the part of school administrators, PTA, school board and teacher groups will be required for discussing the various aspects of the evaluation scheme. The questions to be answered would include:

1. What are the basic objective of the school district (e.g., mainly college preparatory, mainly socialization, etc.)?
2. What are the primary functions needed by the school district in order to attain these objectives (e.g., administrators, teachers, teacher aids, etc.)?
3. What factors are considered necessary or part of the job for each of the job functions in the district (e.g., education, responsibility, subject matter training)?
4. What is the hierarchical structure, in terms of salary, for each of the job functions in the school district (e.g., administrators being paid more than teachers, who are paid more than teacher aids, etc.)?
5. What characteristics in decreasing relative importance constitutes each of the factors (e.g., the factor education should have PhD, MA, BA as characteristics in decreasing relative importance)?

The essential characteristic of any job-evaluation scheme is consistency. This is especially true in a politically sensitive area such as education. It should be stressed that all parties concerned (school board, teacher groups, PTA, administration) with teacher salaries should be included in these preliminary discussions in order to insure the successful implementation of this evaluation scheme in a school district.

The linear programming position-salary evaluation scheme is not limited by the number of factors or functions which might be considered in classifying the certified personnel in a school district. Essentially, the linear programming approach calculate relative weights for each factor, such that the ranking of functions (positions) by salary corresponds to the ranking of the function in the school district salary hierarchy. The resulting analysis and solution of the model then provides the school administration with some measure of the relative importance of each factor for the various functions (positions) within the school district.

IV. DEVELOPING THE POSITION-EVALUATION SCHEME

Metzger[10] suggests five phases in the development of an effective job evaluation scheme by use of linear programming. These phases are:

1. The determination of the factors to be included in the analysis.

   This can usually be accomplished in a school district by an initial survey of the personnel involved in the evaluation, followed by several meetings which are intended to crystallize and result in agreement upon all the factors to be used in the analysis. Some factors to be considered in the evaluation will probably reflect compromise situations, but this is not a major problem to the evaluation scheme. One important result of this phase of the study will be the improvement of ambiguous or poorly written position descriptions.

2. Development of a rating system for the characteristics in each factor.

   A relative rating for each characteristic of each function is defined. For example, the factor education might consist of the following characteristics

Education Factor

Rating	Characteristic
5	PhD or EdD
4	MA or MS
3	MEd
2	BA or BS
1	AA or Junior College
0	H.S. Graduate

The factor education is relatively objective since each characteristic is directly measurable by the completion of a college degree.

A somewhat subjective factor in a school district might concern itself with the difficulty of the learning situation of the school within the school district.

A suggested rating scheme for this factor would be:

Learning Situation Factor

Rating	Characteristic
3	Difficult - disadvantaged area
2	Medium or average difficulty
1	Not difficult - culturally advantaged

The above rating system might present some problems in implementation for many school districts. Since the federal government has developed indices for determining of a given school is culturally disadvantaged, this type of index could be used for this factor. Other partial measures of the difficulty of the learning situation include percent drop out, percentage minority group, discipline or police records of the students. A dichotomous classification (difficult - not difficult) could also be used for this factor. An important concept to remember is that a factor with too few ratings or characteristics will not sufficiently discriminate among deficiencies in ability whereas too many will result in ambiguity. Notice the establishment of the ratings for each characteristic of each function is usually based upon the job description and the mutually-agreed-upon order of relative importance of the characteristics to the particular factor.

3. Determination of the relative weights for each factor in the evaluation.

The determination of the weights to be assigned to each factor is accomplished by means of the model. The model will yield a point system for a number of key functions (positions) in the school district. The rest of the positions in the school district will be determined by the bounds established by these key positions a key position might be considered to be a position where a pay differential has been established, such as between administration and teachers. These key positions usually correspond to the different functions of the personnel in the school district hierarchy.

4. Determination of the remaining salaries in the school district hierarchy.

The relative weights, determined by the solution of the model, are used to evaluate and position, in a hierarchical manner, the remaining positions in the school district. If the salaries for certain positions seem out of place with established school district policies in terms of salary, then either the mathematical formulation of the evaluation scheme must be revised, with more key positions included in the model, or else a significant factor (or factors) was omitted in the analysis of the position.

5. Evaluation of the certified personnel in the school district, his position and his salary.

The primary benefit of the proposed salary evaluation scheme for school districts is the effective evaluation of the certified school district personnel, his position in the hierarchy of the district and his salary. The approach to position-salary evaluation in school administration offers the school administration an internally consistent salary structure which could be of important use in wage-salary negotiations.

V. ADDITIONAL REFINEMENTS TO THE POSITION-EVALUATION SCHEME

In addition to the above phases of the study, the school district may desire to:

1. Establish the relative importance of factors among other factors.

A school district may desire to place heavier emphasis upon a factor such as the difficulty of the learning situation or subject matter preparation. The establishment of the relative importance of the factors or a combination of factors can be established by an ordering of the factors or a relative rating of the factors. This procedure would be very similar to the relative ordering of characteristics within a factor.

2. Investigate other environmental constraints.

The model will assign weights to the various factors in the model based upon the objective function or the criteria of effectiveness established by the school district, subject to the constraint set. In addition to the constraints on salary, which will establish a hierarchical ordering of positions, the evaluation model would include budgetary constraints and percentage relationship concerning salary spreads both within the various functions and between the functions.

3. Determine or develop various objective functions or measures of effectiveness for the evaluation model.

The position-salary evaluation model can be solved for various objective functions. These might include:

(a) the maximization of the beginning teachers salary
(b) the maximization of the salaries for teachers where demand exceeds supply
(c) maximization of the weighting factors in the model or a special objective function which establishes priorities in the weighting factors
(d) the maximization or minimization of some other

specially derived or weighted function of one or more of the variables in the model, agreeable to all parties concerned as valid measure of effectiveness of the salary system.

VI. SUMMARY OF THE PROCEDURES FOR DEVELOPING OF A MODEL FOR POSITION-SALARY EVALUATION FOR A SCHOOL DISTRICT

1. The identification of the certified school district personnel by function or position in the school district salary hierarchy.
2. The identification of those factors which contribute to the performance of the personnel in each function or position in the school district organization.
3. The identification of the descriptors or characteristics within each factor, with an ordering by relative importance.
4. The formulation of the mathematical equations which represent the lowest and highest paid school-district personnel for each function in the school district salary hierarchy.
5. The inclusion into the model of other environmental constraints. These constraints might reflect the "financial environment" of the school district and the various inter and intra position percentage spreads in salary.
6. The determination of the objective function or measure of effectiveness to be employed by the school district in the evaluation scheme.

VII. APPLICATION OF THE MODEL TO A SCHOOL DISTRICT SALARY SCHEDULE

1. Identification of school district personnel by function.

For illustrative purposes only, five functions or positions in a school district were considered in the model. These were:

(a) Superintendent

(b) Administrator (principals, vice principals, etc.)

(c) Department Heads

(d) Teachers

(e) Teacher aids or other para professional personnel such as, lab assistants, readers, etc.

Obviously, other positions such as school nurses, counselors psychometrists, etc., could be included in any direct application of the proposed model to a school district. For simplicity, however, the model will be limited to the five abovementioned functions only.

Determination of Relevant Factors to the Various Functions

After each of the functions in the school district are defined, they each will be evaluated according to nine factors. These are:

1. Area in which the school is located (X_1) to determine teaching environment or relative difficulty of the educational situation.

Rating	Description or Characteristic
3	Disadvantaged
2	Medium or Normal
1	Advantaged

2. Subject matter area (X_2) taught by the teachers in the school district.

Rating	Description
3	Subject areas of high demand compared to available supply; such as, English, math, science, etc.
2	Subject areas of average demand compared to available supply such as Latin, French, etc.
1	Subject areas of low demand compared to available supply (men physical education, social studies, etc.)

3. Supervisory responsibilities of the personnel in the district (X_3) in terms of his areas of responsibility.

Rating	Description
5	District-wide responsibility
4	School unit-wide
3	Department-wide
2	Classroom-wide
1	Student or none

4. Highest degree attained (X_4) at the time of the evaluation by the personnel in the school district.

Rating	Description
5	Ph.D. or Ed.D.
4	M.A. or M.S.
3	M.Ed.
2	B.A. or B.S.
1	AA

5. The total work experience (X_5) of the personnel in the district, plus work years credit for work in previous organization or school districts.

Rating	Description
7	Over 14 years
6	12
5	10
4	8
3	6
2	4
1	2
0	1

6. A factor which allows the district to pay extra remuneration for distinction or awards on the part of its personnel such as Phi Beta Kappa, teaching awards, etc. (X_6).

Rating	Description
2	With distinction
1	With distinction

7. The number of hours per week devoted to school district work or function in addition to the normal work load (X_7).

Rating	Description
5	over 6 hr/week
4	5
3	4
2	3
1	2

8. The number of college units or semester hours completed beyond the highest attained degree (X_8).

Rating	Description
6	over 24
5	over 16
4	over 12
3	over 8
2	over 4
1	over 1

9. The number of in-service hours credit (programs of professional growth) obtained by the teacher or administrator in programs of professional development per year (X_9).

Rating	Description
5	over 10
4	over 8
3	over 6
2	over 4
1	over 2

The above nine factors should not be considered to be a complete list of all factors which might be relevant to the evaluation of school district personnel. Factors such as studies and achievement, organizational climate, workload established by the school administration might also be considered. These factors would be difficult to quantify, but possibly some mutually agreeable system might be established by the various groups involved in the evaluation in order to include these

and other factors in the evaluation model. Suffice it to say, once the factors and their characteristics with their relative ratings are specified it will be possible to describe each function or position in the organizational hierarchy by means of two equations. One equation representing the highest salary paid to the most highly qualified person for that function and the other representing the lowest salary to be received by a person having the minimum of qualifications.

For example, the salary a superintendent in a school district possessing the highest rated characteristics in each of the appropriate factors might be represented by means of the following equation.

$$3X_1 + 5X_3 + 5X_4 + 7X_5 + 2X_6 + X_7 + X_8 + X_9 \leq \lambda$$

Notice factor X_2 (academic subject preparation) is not appropriate in his evaluation.

The salary of a superintendent possessing the lowest rated characteristic (minimum qualifications) in each of the appropriate factors might be represented by an equation of the following from

$$X_1 + 5X_3 + 3X_4 + 2X_5 + X_6 + X_7 + X_8 + X_9 \geq \sigma_1$$

The highest (λ_j) and lowest (σ_j) salaries for each key function j (position) j in the school district salary hierarchy established the framework for the position salary evaluation scheme. In generalized mathematical terms the school district salary hierarchy will take the following from:

$$\alpha_1 X_1 + \alpha_2 X_2 + \ldots + \alpha_n X_n \leq \lambda_j$$

$$\beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_n X_n \geq \sigma_j$$

where

α_i = the highest rated characteristics associated with the factors appropriate to job function j in the school district

β_i = the lowest rated characteristics associated with the factors appropriate to job function j

X_i = the factors associated with job function j

λ_j = the highest or maximum salary to be paid in job classification j

σ_j = the lowest or minimum salary to be paid in job classification j

n = the number of factors considered in the evaluation scheme i = 1, N

Formulation of the Model

With the above generalized equations as a framework for the model representing salary hierarchy of a school district, it is possible to derive a specific set of equations for a school district. One set of equations might take the following form:

Superintendent (highest qualified)

$$3X_1 + 5X_3 + 5X_4 + 7X_5 + 2X_6 + 5X_7 + X_8 + 5X_7 \leq \lambda_1$$

(lowest qualified)

$$X_1 + 5X_3 + 3X_4 + 2X_5 + X_6 + X_7 + X_8 + X_9 \geq \sigma_1$$

Other administrators (highest qualified)

$$3X_1 + 4X_3 + 5X_4 + 7X_5 + 2X_6 + 5X_7 + X_8 + 5X_9 < \lambda_2$$

(lowest qualfied)

$$X_1 + 4X_3 + 3X_4 + X_5 + X_6 + X_7 + X_8 + X_9 \geq \sigma_2$$

Department Head (highest qualified)

$$3X_1 + 3X_2 + 3X_3 + 5X_4 + 7X_5 + 2X_6 + 5X_7 + X_8 + 5X_9 \leq \lambda_3$$

(lowest qualified)

$$X_1 + X_2 + X_3 + 2X_4 + X_5 + X_6 + X_7 + X_8 + X_9 \geq \sigma_3$$

Teacher (highest qualified)

$$3X_1 + 3X_2 + 2X_3 + 5X_4 + 7X_5 + 2X_6 + 5X_7 + 5X_8 + 5X_9 \leq \lambda_5$$

(lowest qualified)

$$X_1 + X_2 + 2X_3 + 2X_4 + X_5 + X_6 + X_7 + X_8 + X_9 \geq \sigma_4$$

Teacher Aid (highest qualified)

$$3X_1 + 3X_2 + X_3 + X_4 + 3X_5 + 2X_6 + 5X_7 + 5X_8 + 5X_9 \leq \lambda_5$$

(lowest qualified)

$$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 \geq \sigma_5$$

Specification of Other Constraints on the System

In addition to the above set of constraints, certain environmental constraints will be included in the model. These are:

1. Setting upper and lower salary bounds on the salary schedule.

λ_1 = ρ the specified maximum salary in the school district, i.e., what you would expect to pay the most highly qualified personnel having the highest position in the organizational hierarchy

σ_5 = the minimum salary in the school district hierarchy

2. Percentage relationships between the highest and lowest salary levels for each classification.

$$\sigma_1 \geq \gamma_1 \lambda_1$$

$$\sigma_2 \geq \gamma_2 \lambda_2$$

$$\sigma_3 \geq \gamma_3 \lambda_3$$

$$\sigma_4 \geq \gamma_4 \lambda_4$$

$$\sigma_5 \geq \gamma_5 \lambda_5$$

where

γ_1 = percentage spread in superintendents salaries

γ_2 = percentage spread in other administrators salaries

γ_3 = percentage spread in department heads salaries

γ_4 = percentage spread in teachers salaries

γ_5 = percentage spread in teacher aids salaries

3. The differences in salaries between the highest salaried personnel in each job classification

$$\lambda_1 - \lambda_2 \leq \delta_1$$

$$\lambda_2 - \lambda_3 \leq \delta_2$$

$$\lambda_3 - \lambda_4 \leq \delta_3$$

$$\lambda_4 - \lambda_5 \leq \delta_4$$

where

δ_1 is the specified difference in salary between the highest paid superintendent and the highest paid administrator

δ_2 is the specified difference in salary between the highest paid administrator and the highest paid department head

δ_3 is the specified difference in salary between the highest paid teacher and highest paid teacher aid

4. The budgetary constraints imposed upon the school district salary schedule.

$$\Sigma n_{ik} \eta_{ik} X_i \leq \Psi$$

n_{ij} = the number of certified employees having characteristic j of factor i

X_i = factor i used in the evaluation scheme

η_{ik} = the relative rating given to characteristic k in factor i

Ψ = the total amount of school district funds available for distribution for certified personnel salaries

5. The total amount of funds available for certified personnel salaries must be maintained at some minimum percentage of the total budget

$$\Psi \geq \omega$$

where

Ψ = the total amount of funds available for teacher salaries

ω = the percentage of the total budget which must be used for certified personnel salaries

φ = the total operating budget of the school system

Determination of the Objective Function

The objective function of the position-salary evaluation model might be to maximize the sum of the weighting factors, subject to the constraints. If the school administrators desire equal weightings to all the factors then the objective function takes the form

$$\text{maximize} \quad \sum_{i=1}^{n} X_i$$

n = the number of factors

If the school district desires to rank or rate each of the factors, so the salary schedule more closely reflects the needs of the district, a utility function of factors may be derived. For example, the school district might desire to rate the factors in the evaluation model in the following manner:

	Rating	Factor
(equal weight)	4	highest degree attained
	4	difficulty of the learning situation
	3	experience
(equal weight)	3	subject matter area
	3	supervisory ability
	3	distinction
(equal weight)	2	other responsibilities
	2	inservice units or credits
	1	units or credits in additional college work

The objective function would take the form

$$\text{maximize } \Sigma W_i X_i$$

where

X_i = factor i

W_i = the rating given to factor i

By using the abovementioned objective function, greater financial "rewards" will be given to those certified personnel processing the higher rated characteristics in the higher rated factors. This would mean that school personnel possessing higher degrees and working in disadvantaged areas could receive the higher salaries within their position in the hierarchy.

In addition to the previously mentioned objective functions, the school administration might desire to minimize total salary costs for certified personnel (Ψ) or maximize the beginning teacher salary (σ_4) or develop, in conjunction with various groups involved in the evaluation, more sophisticated objective functions or utility functions which would better reflect the desired criteria of effectiveness of the salary system in the school districts.

VIII. SUMMARY OF THE POSITION SALARY EVALUATION MODEL

Hierarchical Constraints

Superintendent:

$$\begin{cases} 3X_1 + 5X_3 + 5X_4 + 7X_5 + 2X_6 + 5X_7 + X_8 + 5X_9 \leq \lambda_1 \\ X_1 + 5X_3 + 3X_4 + 2X_5 + X_6 + X_7 + X_8 + X_9 \geq \sigma_1 \end{cases}$$

Other Administrators:

$$\begin{cases} 3X_1 + 4X_3 + 5X_4 + 7X_5 + 2X_6 + 5X_7 + X_8 + 5X_9 \leq \lambda_2 \\ X_1 + 5X_3 + 3X_4 + X_5 + X_6 + X_7 + X_8 + X_9 \geq \sigma_2 \end{cases}$$

Department Heads:

$$\begin{cases} 3X_1 + 3X_2 + 3X_3 + 5X_4 + 7X_5 + 2X_6 + 5X_7 + X_8 + 5X_9 \leq \lambda_3 \\ X_1 + X_2 + 3X_3 + 2X_4 + X_5 + X_6 + X_7 + X_8 + X_9 \geq \sigma_3 \end{cases}$$

Teachers:

$$\begin{cases} 3X_1 + 3X_2 + X_3 + X_4 + 3X_5 + 2X_6 + 5X_7 + 5X_8 + 5X_9 \leq \lambda_4 \\ X_1 + X_2 + 2X_3 + 2X_4 + X_5 + X_6 + X_7 + X_8 + X_9 \geq \sigma_4 \end{cases}$$

Teacher Aides:

$$\begin{cases} 3X_1 + 3X_2 + X_3 + X_4 + 3X_5 + 2X_6 + 5X_7 + 5X_8 + 5X_9 \leq \lambda_5 \\ X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 \geq \sigma_5 \end{cases}$$

Maximum and Minimum Salary Level Constraints for the District

$$\lambda_1 \leq \rho$$

$$\sigma_5 \geq \mu$$

Percentage Relationship Constraints Governing the Spread in Salaries Within Each Classification

$$\sigma_1 \geq \gamma_1 \lambda_1$$

$$\sigma_2 \geq \gamma_2 \lambda_2$$

$$\sigma_3 \geq \gamma_3 \lambda_3$$

$$\sigma_4 \geq \gamma_4 \lambda_4$$

$$\sigma_5 \geq \gamma_5 \lambda_5$$

Dollar Spread Constraint in Salary Between the Highest Paid Personnel in Each Position Classification

$$\lambda_1 - \lambda_2 \leq \delta_1$$

$$\lambda_2 - \lambda_3 \leq \delta_2$$

$$\lambda_3 - \lambda_4 \leq \delta_3$$

$$\lambda_4 - \lambda_5 \leq \delta_4$$

Budgetary Constraints on the System

$$\Sigma\, n_{ik} \eta_{ik} X_i \leq \Psi$$

$$\Psi = \varphi \Phi$$

Objective function:

maximize

$$\Sigma\, w_i X_i$$

where

W_i = the relative rating given to each factor i

ρ = maximum salary in the district

μ = minimum salary in the district

X_i = relative weight for factor i

λ_j = maximum salary for each job classification j in the school district

σ_j = minimum salary for each job classification j in the school district

δ_j = dollar spread in salary between the highest paid personnel in each of the job classifications j and j+1 .

γ_j = percentage spread in salary between the highest and lowest paid personnel in each job classification j

n_{ik} = the number possessing each characteristic k of each factor i

Ψ = the maximum amount of school district funds available for certified salaries

φ = minimum percentage of the total budget available for certified salaries

η_{ik} = the rating given to characteristic k in factor i

Φ = the total school district budget

IV. USE OF THE MODEL WITH THE POST-OPTIMAL-SENSITVITY ANALYSIS FOR FUTURE PLANNING

Once the salary-evaluation model is formulated and objective function determined, the school administrator will possess a relatively sophisticated method for calculating the salary level for the school district personnel included in the model. The constraint set of the model will insure that all salaries are consistent with hierarchical ordering of the school district salary schedule and the environmental constraint set. If a school district desires to test the cost sensitivity of the model to changes in the constraint set, this can be accomplished by the parameterization of selected variables or by the post-optimal sensitivity analysis, which includes the dual solution, the reduced costs and the ranging analysis. The dual solution informs the decisionmaker of the resulting consequences in the value of the objective function or effectiveness of the system resulting from

a one unit relaxation of each constraint now in the model. The analysis of the reduced costs informs the decisionmaker of the resulting consequences in the value of the objective function if an added unit of activity for each variable (separately) is forced into the solution. The ranging analysis informs the decisionmaker of the values of the upper and lower bounds (range) of values, for both the right-hand side constraint and the coefficients of the objective function, which the solution will remain optimal. The type of post-optimal analysis can give the decisionmaker valuable insights into the sensitivity of the constraint set of the model and indicate areas where more information or study is needed and suggest strategies for future refinements of the model.

X. SUMMARY

The linear programming approach to position-salary evaluation of school district personnel allows the school district to calculate a system of relative weights, which establishes the relationship of one position to another, in quantitative terms. It also allows the school district to establish a consistent salary difference between the various positions or functions. In summary the proposed approach to school district wage and salary administration has the following characteristics and advantages which distinguish it from other salary evaluation schemes in education. These are:

First, an internally consistent evaluation scheme which is valid for all the functions of school personnel considered in the model and which takes into consideration all the agreed-upon factors which constitute those functions.

Second, the model presents to the school district a more effective assessment of the individual's relative worth to the school district in terms of salary.

Third, the model could be used to justify salary increases in school district personnel and play an important role in wage-salary negotiations with teacher unions and associations.

Fourth, establishes a salary hierarchy consistent with the objectives of the school district, but allows for highly qualified personnel in one function to receive larger salaries than the lowest qualified personnel in a higher functions (e.g., highly qualified teachers could receive higher pay than low qualified administrators).

Fifth, allows the school district to establish salary priorities, (e.g., a school district can pay higher salaries to teachers in difficult learning areas, or pay higher salaries for teachers in high demand low supply teaching areas).

Sixth, encourages participation of teacher groups, administrators, PTA, and school board in setting the objectives of the school, the functions or job descriptions of the personnel in the school and finally the establishment of those factors necessary to perform the particular function along with a rating of the characteristics which constitute each factor.

The utilization of linear programming approach to position-salary evaluation in school districts could be defended on theoretical grounds because:

1. School districts are typically more concerned with internal rather than external salary relationships. That is teachers are usually in a better position to evaluate his contribution and his salary relative to others in the school districts than to those with comparable positions in other school districts.

2. The linear program technique has the ability to compensate for internal inequities that may already exist within an organization. Thus if several positions in the organization are out of line in terms of salary, they will be identified because of the internal consistency of the model.

XI. CONCLUSION

In conclusion, a linear programming approach to position-salary evaluation can be of both practical and theoretical importance in school personnel administration. Such models would not only provide a relatively sophisticated method for determining the most efficient allocation of resources consistent with the objectives of the school district and subject to the constraints, but can be used to place

greater emphasis (salarywise) and assign greater importance to those factors and areas which are considered essential by the school district. The involvement of teacher groups, administrators, PTA and school board in the development of the model will not only add to the validity of the model, but increase communication and understanding of school district problems. Finally, the determination of the relative weights to be assigned each of the factors, consistent with the budgetary and hierarchical constraints imposed on the system, and maximizes or minimizes the specified objective function, will form the basis for an internally consistent, fair and rational position-salary evaluation scheme for school districts.

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